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Empirical modeling of high-income and emerging stock and Forex market return volatility using Markov-switching GARCH models

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ABSTRACT

Using weekly data for stock and Forex market returns, a set of MS-GARCH models is estimated for a group of high-income (HI) countries and emerging market economies (EMEs) using algorithms proposed by Augustyniak (2014) and Ardia et al. (2018, 2019a,b), allowing for a variety of conditional variance and distribution specifications. The main results are: (i) the models selected using Ardia et al. (2018) have a better fit than those estimated by Augustyniak (2014), contain skewed distributions, and often require that the main coefficients be different in each regime; (ii) in Latam Forex markets, estimates of the heavy-tail parameter are smaller than in HI Forex and all stock markets; (iii) the persistence of the high-volatility regime is considerable and more evident in stock markets (especially in Latam EMEs); (iv) in (HI and Latam) stock markets, a single-regime GJR model (leverage effects) with skewed distributions is selected; but when using MS models, virtually no MS-GJR models are selected. However, this does not happen in Forex markets, where leverage effects are not found either in single-regime or MS-GARCH models.

1. Introduction

Financial market volatility plays an important role in economic performance and financial stability. In particular, the specification of conditional volatility is essential for constructing risk measures; see Ardia (2008). Furthermore, modeling time-varying volatility has been widely used in the literature on financial time series, as the demand for monitoring volatility has increased as a means of assessing financial risk. Two approaches that have proved useful are the autoregressive conditional heteroskedasticity (ARCH) family, including the ARCH model developed by Engle (1982); the generalized ARCH (GARCH) model by Bollerslev (1986); and the

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stochastic volatility (SV) model introduced by Taylor (1982) and further developed by Taylor (1986).[3] Multiple extensions of these models have been proposed to capture additional stylized facts observed in financial series, such as non-linearities, asymmetries, and long memory.[4] Another characteristic of the return distribution of financial (stock) series is the asymmetric response of volatility, known as the leverage effect, first noted by Black (1976) and modeled by Nelson (1991) and Glosten, Jagannathan, and Runkle (1993)-GJR, among others.

However, many financial series exhibit structural changes in the dynamics of volatility. In these circumstances, volatility modeling and predictions made using GARCH-type models fail to fully capture volatility movements; see Lamoureux and Lastrapes (1990), Danielsson (2011) and Bauwens, De Backer, and Dufays (2014). One way to deal with this problem is allowing the parameters of the GARCH model to vary according to a latent variable that follows a Markov process (see Hamilton, 1989), which in turn gives rise to the MS-GARCH model. This specification allows a different GARCH behavior in each regime; i.e., it is possible to capture the difference in variance dynamics both in periods of low and high volatility.

Initial studies about MS models applied to financial time series focused on ARCH-type specifications; see Cai (1994) and Hamilton and Susmel (1994). Excluding lagged values of the conditional variance in the variance equation allows the likelihood function to be computationally treatable. When using a GARCH-type specification, since there is a Markov chain with K regimes, assessing the likelihood requires the integration of all K^T possible paths, which makes the estimation unfeasible. Gray (1996) and Dueker (1997), and Klaassen (2002) first attempted to address this issue, known as the path-dependence problem. Essentially, they tackle the problem by collapsing the past regime-specific conditional variances using particular schemes. For instance, Gray (1996) suggests that the conditional distribution of returns is independent of the regime path; and integrates the path of the regime not observed in the GARCH equation through the conditional expectation of the past variance. Others suggest alternative estimation methods to face the problem of path dependence without modifying the MS-GARCH model.[5]

Recently, Augustyniak (2014), hereinafter AGK, estimates an MS-GARCH model using Monte Carlo Expectation Maximization (MCEM) and Monte Carlo Maximum Likelihood (MCML) algorithms, and obtains an approximation of the asymptotic standard errors of the maximum likelihood estimates. AGK finds that the MCEM-MCML algorithm is effective in the simulation of the posterior distribution of the state vector in empirical results using daily and weekly S&P500 price index returns. Another recent contribution is Ardia, Bluteau, Boudt, and Catania (2018), hereinafter ABBC, who consider an alternative approach, suggested by Haas, Mittnik, and Paolella (2004), which consists in letting the GARCH process of each regime evolve independently from the other states. While this approach avoids the path-dependence problem, it has the additional advantage of allowing more clarity in the interpretation of variance dynamics in each regime. ABBC implement their different models using the MS-GARCH R Package of Ardia, Bluteau, Boudt, and Catania (2019a) and Ardia, Bluteau, Boudt, Catania, and Trottier (2019b). Thus, they estimate a wide variety of models that support several specifications (e.g., GARCH and Glosten et al. (1993)) with different types of innovations. They apply these models to the prediction of different risk measures; e.g., value-at-risk (VaR) and expected shortfall; and find that the MS-GARCH models offer better results compared to different single-regime GARCH/GJR specifications. See also Iqbal (2016) and Abounoori, Elmi, and Nademi (2015) about forecasting volatility and risk measures in the Karachi and Tehran stock markets, respectively.

The literature includes other contributions in addition to the empirical applications of AGK, ABBC, and Ardia et al. (2019a, 2019b). Moore and Wang (2007) analyze stock market volatility in five new states of the European Union (the Czech Republic, Hungary, Poland, Slovenia, and Slovakia) in 1994–2005. The results reveal a tendency to low market volatility in these markets when they joined the European Union compared with the previous high-volatility period. Liang and Yongcheol (2008) apply MS-GARCH models to weekly data from five emerging market economies (EMEs) in East Asia; and other similar stock market studies are Marcucci (2005), Wang and Theobald (2008), Visković, Arnerić, and Rozga (2014), Lolea and Vilcu (2018) and Korkpoe and Howard (2019). Ardia, Bluteau, and Rüede (2019c) find strong evidence of regime changes in the GARCH dynamics of volatility in the bitcoin market; i.e., MS-GARCH models outperform single-regime specifications when predicting VaR. López-Herrera and Mota (2019) analyze the relationship between Mexican stock market yields and USD yields (i.e., the appreciation rate), as well as the relationship between their volatilities using MS-GARCH models. They find evidence suggesting an association between stock market returns and the appreciation/depreciation of the domestic currency and a positive association when volatility is high in both markets. Oseifuah and Korkpoe (2018) use an MS-GARCH model with skewed Student-t innovations to examine the exchange rate dynamics in South Africa (relative to the dollar) and find evidence of secular changes in the South African economy that push the domestic currency into a dominant high-volatility regime. Other studies on the Forex market are Klaassen (2002), Sopipan, Intarasit, and Chuarkham (2014), Caporale and Zekokh (2019), and Hamida and Scalera (2019). Other documents with empirical applications of MS-GARCH models are Gray (1996), who analyzes changes in short-term interest rates; Billio, Casarin, and Osuntuyi (2018) and Günay (2015), who focus on the dynamics of the energy futures markets and on modeling the volatility of oil returns, respectively; and Allen et al. (2013), who study the dynamics of hedging in energy futures markets.

[3] For extensive reviews, see Bollerslev, Engle, and Nelson (1994) and Engle (1995) for the ARCH family models, and Shephard (2005) for a comprehensive explanation of the SV models.

[4] For comprehensive reviews, see Franses and van Dijk (2000), Engle (2004) and Teräsvirta (2009). For stylized facts about Peru's stock and Forex markets, see Humala and Rodríguez (2013). For the long-memory property and other stylized facts in Latam stock and Forex markets, see Rodríguez (2016) and Rodríguez (2017) and the references mentioned therein.

[5] Francq and Zakoian (2008) use the generalized method of moments (GMM) with the analytical expressions of Francq and Zakoian (2005), whereas Bauwens, Preminger, and Rombouts (2010) are the first to use Bayesian MCMC techniques to estimate the MS-GARCH model, providing sufficient conditions for geometric ergodicity and the existence of moments in the process.

This paper seeks to contribute to the empirical literature by modeling and analyzing volatility in stock and Forex markets for a group of high-income (HI) countries and Latam EMEs. The HI countries chosen are Canada, the U.S., Denmark, Norway, Australia, Switzerland, the UK, Japan, and Europe. The Latam EMEs are Argentina, Brazil, Chile, Colombia, Mexico, and Peru. Taking into account the sample of markets and countries, as well as a broad set of single-regime GARCH/GJR and MS-GARCH/MS-GJR models, this document has the following objectives: estimating and analyzing the behavior of a high-volatility regime while identifying the events associated with stress periods; calculating the persistence of this regime; and identifying the presence of biases, heavy tails, and leverage effects according to the distributions selected for the estimations. Selection of the best models is done taking into account several criteria: value of the log-marginal likelihood; significance of the parameters; and evaluation of the smoothed curve of probabilities for the high-volatility regime associated with the correct identification of the main domestic and international events that create stress in volatility episodes.

To our best knowledge, this is the first comparative work between a diverse group of HI countries and EMEs, as well as a comparison between the stock and Forex markets using a wide variety of single-regime GARCH/GJR and MS-GARCH/MS-GJR models with different innovations. The main results are: (i) the models selected using Ardia et al. (2018) have a better fit than those estimated by Augustyniak (2014), contain skewed distributions, and often require the main coefficients to be different in each regime; (ii) estimates of the heavy-tail parameter in Latam Forex markets is smaller than in HI Forex markets and in all stock markets; (iii) the persistence of a high-volatility regime is high and more evident in stock markets (especially in Latam EMEs); (iv) in (HI and Latam) stock markets, a single-regime GJR model (leverage effects) with skewed distributions is selected; but when using MS models, virtually no MS-GJR models are selected. However, this does not happen in Forex markets, where leverage effects are not found either in single-regime or MS-GARCH models.

The rest of the paper is organized as follows. Section 2 presents the different models used in this paper. Section 3 describes and analyzes the data and shows the empirical results of the models. The conclusions are presented in Section 4.

2. Methodology

In order to abbreviate and simplify the presentation, we assume that the log-returns have a zero mean and are not autocorrelated,[6] and this variable is denoted by r_t. Four types of models are presented below: the single-regime GARCH(1,1) model with Normal innovations; the MS-GARCH(1,1) model by AGK; the MS-GARCH(1,1) and MS-GJR(1,1) models used by ABBC; and the single-regime GARCH(1,1) and the single-regime GJR(1,1) models with alternative distributional specifications. The estimation of an MS-GARCH model suffers from the so-called path dependence problem, which causes serious estimation difficulties. The first attempt to solve this problem was Gray (1996), but we follow the more efficient approach of AGK. Notice that the approach of ABBC, following Haas et al. (2004), does not address this problem.

2.1. The Generalized ARCH (GARCH) Model

The GARCH(1, 1) model of Bollerslev (1986) can be written as:

$$r_t = \sqrt{h_t}\,\eta_t \tag{1}$$

$$h_t = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 h_{t-1} \tag{2}$$

where $\eta \sim i.\ i.\ d.\ \mathcal{N}(0, 1)$, $\alpha_0 > 0$, $\alpha_1 \geqslant 0$ and $\beta_1 \geqslant 0$ to ensure a positive conditional variance h_t, and $\alpha_1 + \beta_1 < 1$ to ensure that the unconditional variance $h_t = \alpha_0/(1 - \alpha_1 - \beta_1)$ is defined.

2.2. The MS-GARCH model of Augustyniak (2014)

Based on Bauwens et al. (2010) and Francq, Roussignol, and Zakoian (2001) and using AGK notation, the MS-GARCH model can be defined by the following equations:

$$r_t(s_t) = \sqrt{h_t(s_{1:t})}\,\eta_t, \tag{3}$$

$$h_t(s_{1:t}) = \alpha_{0,s_t} + \alpha_{1,s_t} r_{t-1}^2(s_{t-1}) + \beta_{1,s_t} h_{t-1}(s_{1:t-1}), \tag{4}$$

where $\eta \sim i.\ i.\ d.\ \mathcal{N}(0, 1)$. At each point in time, the conditional variance is $h_t(s_{1:t}) = \mathrm{var}(r_t|r_{1:t-1}, s_{1:t})$, where $r_{1:t-1}$ and $s_{1:t}$ are shorthand for the vectors $(r_1, \ldots, r_{t-1})$and $(s_1, \ldots, s_t)$, respectively. The process $\{s_t\}$ is an unobserved ergodic time-homogeneous Markov chain process with K-dimensional discrete state space (i.e., s_t can take integer values from 1 to K). The $K \times K$ transition matrix of the Markov chain is defined by the transition probabilities $\{\Pr[s_t = j|s_{t-1} = i] = p_{ij}\}_{i,j=1}^{K}$. The vector $\theta = (\{\alpha_{0,i}, \alpha_{1,i}, \beta_{1,i}\}_{i=1}^{K}, \{p_{ij}\}_{i,j=1}^{K})$ denotes the parameters of the model. In order to ensure positivity of the variance, the following constraints are required: $\alpha_{0,i} > 0$, $\alpha_{1,i} \geqslant 0$ and $\beta_{1,i} \geqslant 0, \quad i = 1, \ldots, K$. Conditions for stationarity and the existence of moments are studied by Bauwens et al. (2010), Francq et al. (2001) and Francq and Zakoian (2005).

It is worth highlighting that the notation used in (3–4) emphasizes the dependence of the conditional variance at time t on the

[6] In practice, it means that we apply the models using demeaned log-returns, as explained in Section 3.

entire regime path $s_{1:t}$, which is the path dependence issue. Furthermore, AGK imposes the restrictions $\alpha_{1,1} = \alpha_{1,2}$ and $\beta_{1,1} = \beta_{1,2}$ on two alternative MS-GARCH models to find results consistent with empirical evidence.

2.3. The MS-GARCH model of Ardia et al. (2018)

We follow the notation of Ardia et al. (2018), Ardia et al., 2019a and Ardia et al., 2019b, where the process of conditional variance is regime-switching dependent. Denote by $\mathscr{I}_{t-1}$ the information set observed up to $t-1$, that is, $\mathscr{I}_{t-1} \equiv \{r_{t-i}, i > 0\}$. In general terms, ABBC express the MS-GARCH model using the following specification:

$$r_t|(s_t = k, \mathscr{I}_{t-1}) \sim \mathscr{D}(0, h_{k,t}, \xi_k), \tag{5}$$

where $\mathscr{D}(0, h_{k,t}, \xi_k)$ is a continuous (conditional) distribution with zero mean, $h_{k,t}$ is the time-varying variance, and the vector ξ_k includes additional shape parameters (e.g., asymmetry, kurtosis). The latent variable s_t is defined in the discrete space $\{1, ..., K\}$, and evolves according to an unobserved first-order ergodic homogeneous Markov chain with a $K \times K$ transition probability matrix $\mathbf{P} \equiv \{p_{ij}\}_{i,j=1}^K$ as defined above. In this context, $\eta \sim i.\ i.\ d.\ \mathscr{N}(0, 1)$, which appears in 2.1 and 2.2, is a special case of $\mathscr{D}(\cdot)$. Furthermore, defining $E[r_t^2|s_t = k, \mathscr{I}_{t-1}] = h_{k,t}$, where $h_{k,t}$ is the variance of r_t conditional on the realization of $s_t = k$ and $\mathscr{I}_{t-1}$, ABBC specify the variance of r_t as a GARCH type model; i.e., conditional on the regime $s_t = k$, we have that $h_{k,t} \equiv h(r_{t-1}, h_{k,t-1}, \theta_k)$. Different specifications for $h(.)$ are proposed by ABBC. We use two of them: (i) the first one is an MS-GARCH (1,1) model as suggested by Haas et al. (2004), that is,

$$h_{k,t} = \alpha_{0,k} + \alpha_{1,k} r_{t-1}^2 + \beta_{1,k} h_{k,t-1},$$

for $k = 1, 2$ an where, to ensure positivity, we require that $\alpha_{0,k} > 0$, $\alpha_{1,k} > 0$, $\beta_{1,k} \geqslant 0$ and to ensure stationarity[7] we require that $\alpha_{1,k} + \beta_{1,k} < 1$. In this case, $\theta_k \equiv (\alpha_{0,k}, \alpha_{1,k}, \beta_{1,k})'$; (ii) the second one is an MS-GJR(1,1) model which exploits the volatility specification of Glosten et al. (1993):

$$h_{k,t} = \alpha_{0,k} + (\alpha_{1,k} + \alpha_{2,k} \mathbb{I}_{\{r_{t-1} < 0\}}) r_{t-1}^2 + \beta_{1,k} h_{k,t-1},$$

for $k = 1, 2$, where $\mathbb{I}_{\{\cdot\}}$ is the indicator function taking a value of one if the conditions hold, and zero otherwise; and where, to ensure positivity, we require that $\alpha_{0,k} > 0$, $\alpha_{1,k} > 0$, $\beta_{1,k} \geqslant 0$ and to ensure stationarity we require that $\alpha_{1,k} + \alpha_{2,k} E[\eta_{k,t}^2 \mathbb{I}_{\{\eta_{k,t} < 0\}}] + \beta_{1,k} < 1$.[8] In this case, $\theta_k \equiv (\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}, \beta_{1,k})'$. The parameter $\alpha_{2,k} \geqslant 0$ controls the degree of asymmetry in the conditional volatility process. This model accounts for the well-known asymmetric reaction of volatility to the sign of past returns, which is frequently referred to as the leverage effect; see Black (1976).

Flexibility in the approach of Ardia et al. (2019a) and Ardia et al., 2019b provides several options for $\mathscr{D}(.)$, of which we use four: (i) the standard Normal distribution ($\mathscr{N}$); (ii) the Student-t distribution ($\mathscr{S}$); (iii) the skewed Normal distribution ($sk\mathscr{N}$); and (iv) the skewed Student-t distribution ($sk\mathscr{S}$). The distributions in (iii) and (iv) are used to assess the benefits of introducing asymmetry in the analysis following the suggestions of Fernández and Steel (1998) and Bauwens and Laurent (2005); see Trotier and Ardia (2016) for further details concerning the derivation of the moments of these standardized distributions.

2.4. Other models

While the single-regime GARCH (1,1)-$\mathscr{N}$ model presented in 2.1 is usually not a good competitor against more flexible specifications, such as MS-type models, other single-regime GARCH models may be better competitors if distributional flexibility of $\mathscr{D}(.)$ is allowed. Thus, we estimate two additional models: a single-regime GARCH(1,1) and a single-regime GJR(1,1) with the distributional specifications mentioned in (ii), (iii) and (iv) in 2.3. Therefore, we also estimate models where $h_t = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 h_{t-1}$ and $h_t = \alpha_0 + (\alpha_1 + \alpha_2 \mathbb{I}_{\{r_{t-1} < 0\}}) r_{t-1}^2 + \beta_1 h_{t-1}$, with distributions $\mathscr{S}$, $sk\mathscr{N}$, and $sk\mathscr{S}$, respectively.

2.5. Estimation of the MS-GARCH model

Given the predominantly empirical nature of this paper, we do not intend to explain the detail the AGK and ABBC estimation methods. For information, a brief sketch is provided below.

To estimate the parameters of the MS-GARCH model, AGK develops a method based on the MCEM algorithm of Wei and Tanner (1990) and the MCML method suggested by Geyer (1994) and Geyer (1996). Since the method uses the posterior distribution of the state variables, it is usually seen as an equivalent frequentist approach to the Bayesian MCMC method proposed by Bauwens et al. (2010). The MS-GARCH model specified by Eqs. 3,4 presents estimation challenges, because the conditional variance t depends on the complete path $s_{1:t}$. The likelihood of the observations, $f(r|\theta)$, is calculated by integrating all possible regime paths. An accurate estimate of the log-likelihood is obtained by Bauwens et al. (2010) by writing $\log f(r_t|\theta) = \log(r_1|\theta) + \sum_{t=1}^{T-1} \log f(r_{t+1}|r_{1:t}, \theta)$ and es-

[7] Covariance-stationarity is a stronger requirement than that of Hass et al. (2004); see ABBC for further details.

[8] In general, covariance-stationarity is achieved by imposing $\alpha_{1,k} + \alpha_{2,k}\kappa_k + \beta_{1,k} < 1$, where $\kappa_k \equiv \Pr[r_{t-1} < 0|s_t = k, \mathscr{I}_{t-1}]$. If $\eta_{k,t}$ is symmetrically distributed, then $E[\eta_{k,t}^2 \mathbb{I}_{\{\eta_{k,t} < 0\}}] = \frac{1}{2}$. For skewed distributions, we follow Trottier and Ardia (2016).

timating $f(r_{t+1}|r_{1:t}, \theta)$, $t = 1...,T - 1$ sequentially with the aid of particle filters. Simulation of the log-likelihood is difficult to maximize with standard optimization routines, because this filter is not a continuous function of θ.[9] The hybrid AGK approach implies a two-step calculation. While in the first step the MCEM algorithm obtains a good MLE estimate, θ^*, in the second step the MCML method replaces many potential iterations of the MCEM with a single iteration, leading to faster convergence. The MCEM-MCML algorithm uses as starting points the approximations of the Gray (1996), Dueker (1997) and Klaassen (2002) models. To initialize the Gibbs sampler, a smoothed inferred state vector, as proposed by Gray (1996), is taken as the first state vector; and to generate the first Markov chain s_1, we assume the initial state s_0 as given (i.e., it is fixed instead of estimated).

On the other hand, ABBC offer two techniques for estimating the parameters of the MS-GARCH models: ML and Bayesian MCMC. In order to compare with AGK, we use the first approach. Let $\Psi \equiv (\theta_k, \mathbf{P})$ be the vector of parameters. The log-likelihood function is $\log\mathbf{L}(\Psi|\mathscr{I}_T) \equiv \sum_{t=1}^{T} \log f(r_t|\Psi, \mathscr{I}_{t-1})$, where $f\left(r_t|\Psi, \mathscr{I}_{t-1}\right) \equiv \sum_{i=1}^{K}\sum_{j=1}^{K} p_{ij} z_{i,t-1} f_{\mathscr{D}}\left(r_t|s_t = j, \Psi, \mathscr{I}_{t-1}\right)$ denotes the conditional density of r_t given past observations, $\mathscr{I}_{t-1}$; and $z_{i,t-1} \equiv \Pr[s_{t-1} = i|\Psi, \mathscr{I}_{t-1}]$represents the filtered probability of state i at time $t - 1$obtained via the filter proposed by Hamilton (1989) and Hamilton (1994). The ML estimator, $\widehat{\Psi}$, is obtained by maximizing the log likelihood function. For more details, see Trottier and Ardia (2016), and ABBC.[10]

3. Empirical evidence

3.1. Data and preliminary statistics

Weekly series for stock and Forex market returns are built for HI countries (Canada, the U.S., Denmark, Norway, Australia, Switzerland, the UK, Japan, and Europe) and Latam EMEs (Argentina, Brazil, Chile, Colombia, Mexico, and Peru) from daily Bloomberg Financial Data. The weekly data are from Wednesday to Wednesday to avoid most public holidays.[11, 12] If p_t denotes a stock market index or an exchange rate,[13] then the percentage log-return series is defined by $y_t = 100 \times [\log(p_t) - \log(p_{t-1})]$, where the index t denotes the weekly closing observations. Then, following ABBC, we demean the returns y_t using an AR(1) filter, and use the filtered returns, denoted by r_t, to estimate all models. The samples for all countries and markets end on July 24, 2019, but the beginning of each sample is different and explained by the availability of observations. In the case of HI stock markets, the U.S., Denmark, Switzerland, Japan, and Europe begin in 1990 (February 26, January 30, January 24, and January 17, respectively). The starting dates of Latam stock market series are as follows: December 25, 1991 (Argentina); March 15, 1995 (Brazil); August 8, 1990 (Chile); July 25, 2001 (Colombia); March 30, 1994 (Mexico); and February 6, 2002 (Peru). All series for HI Forex markets start in January 1990 except Canada (August 26, 1998). The starting dates of Latam Forex markets are as follows: July 2, 1999 (Brazil); January 17, 1990 (Chile); September 2, 1992 (Colombia); May 8, 1996 (Mexico); and May 24, 1995 (Peru). The sample for Argentina begins on March 5, 2014 given presence of fixed exchange rate periods in that country.[14]

Table 1 shows the descriptive statistics for stock and Forex returns. Panels (a) and (b) show information for (HI and Latam) stock markets, respectively. HI markets show extreme values between a minimum of −21.23 (Japan) and a maximum of 20.38 (Norway). Among Latam markets, Brazil and Argentina show extreme values. Chile and Colombia have the smallest maximum values, although they are higher than for Canada and the U.S. Regarding the standard deviation, all Latam markets are more volatile than HI markets. Overall, the most volatile markets are Norway, Japan, Argentina, and Brazil. Additionally, all asymmetry coefficients are negative, with higher values (in absolute values) in Norway, Canada, and the U.S. In general, this coefficient is smaller (in absolute values) in Latam markets. At the same time, Norway shows the widest departure from Normality. Among Latam markets, Brazil shows high kurtosis, followed by Peru, Colombia, and Chile, with values higher than the other HI markets.

Panels (c) and (d) show information about (HI and Latam) Forex markets. In general, the observed minimum values (in absolute values) are lower than in the stock market. The maximum values are also smaller than in the stock markets, except in Norway and Australia among HI markets and Argentina, Brazil, and Mexico among Latam markets. The standard deviation is no greater than 1.6 in HI countries, lower than in stock markets. In Latam markets, Argentina and Brazil show higher values than HI markets, but smaller than Latam stock markets. Peru appears to be the less volatile Forex market. It is important to mention that lower volatility in some Latam markets may be due to central bank Forex market intervention. Additionally, positive and negative asymmetry values are present. The asymmetry coefficients for HI markets are smaller than for their respective stock markets, except in Australia,

[9] Given this deficiency, Gray (1996) proposes replacing the conditional variance $h_{t-1}(s_{1:t-1}) = \text{var}[r_{t-1}|r_{1:t-2}, s_{1:t-1}])$ by $h_{t-1} = \text{var}[r_{t-1}|r_{1:t-2}]$; i.e., collapsing all possible conditional variances at time $t - 1$ into a single value that does not depend on the regime path. However, AGK shows that Gray's method does not generate consistent estimators for the MS-GARCH.

[10] Future research may include estimates using Bayesian MCMC methods. For this paper, given the large number of models that have been estimated (see next Section), this possibility has been ruled out.

[11] Given the omission of Wednesday data for any given week, we decided to choose some other "feasible day"' of the week. The criterion for this choice is based on the construction of a ranking of missing data (from lowest to highest) on each day of the week throughout the daily series, selecting as the "first feasible" the day of the week with fewer omissions. If it did not exist, we selected the following day in the ranking of omissions as a "second feasible" day; and so on. In this way we built weekly series with no missing data.

[12] Weekly data are used due to the presence of more noise in higher frequencies, such as daily data, which makes it more difficult to isolate cyclical variations, thereby obscuring the analysis of driving moments of switching behavior; see for instance Moore and Wang (2007).

[13] The exchange rate is measured as domestic currency units per USD.

[14] This allows only 282 observations to be obtained, which represents between 15% or 20% of the samples from the other Latam Forex markets.

Table 1
Descriptive Statistics for Stock and Forex Markets Returns.

Country	Security ID	Start Date	End Date	Obs.	Std.	Min	Max	Skew	Kurt
			(a) Stock – High Income Countries						
Canada	SPTSX	30-Jan-1991	24-Jul-2019	1487	2.06	−15.31	7.77	−0.73	6.74
USA	SPX	24-Jan-1990	24-Jul-2019	1540	2.17	−16.75	9.58	−0.75	7.88
Denmark	KFX	26-Dec-1990	24-Jul-2019	1492	2.61	−18.83	11.20	−0.59	6.78
Norway	OSEBX	28-Feb-1996	24-Jul-2019	1222	3.08	−20.99	20.38	−0.97	9.42
Australia	AS51	10-Nov-1993	24-Jul-2019	1342	1.96	−12.06	11.58	−0.49	6.17
Switzerland	SMI	24-Jan-1990	24-Jul-2019	1540	2.46	−14.88	14.00	−0.71	7.31
UK	UKX	17-Jan-1990	24-Jul-2019	1541	2.24	−13.05	12.80	−0.37	6.35
Japan	NKY	17-Jan-1990	24-Jul-2019	1541	3.06	−21.23	14.71	−0.40	5.84
Europe	SX5E	17-Jan-1990	24-Jul-2019	1541	2.80	−15.88	15.15	−0.66	6.53
			(b) Stock – Emerging Countries (Latam)						
Argentina	MERVAL	25-Dec-1991	24-Jul-2019	1440	5.02	−22.77	26.43	−0.29	5.15
Brazil	IBOV	15-Mar-1995	24-Jul-2019	1272	4.21	−27.73	31.20	−0.40	10.94
Chile	IPSA	08-Aug-1990	24-Jul-2019	1512	2.76	−21.83	13.03	−0.20	8.39
Colombia	IGBC	25-Jul-2001	24-Jul-2019	940	2.89	−22.11	14.84	−0.68	9.98
Mexico	MEXBOL	30-Mar-1994	24-Jul-2019	1322	3.25	−19.65	17.22	−0.11	6.59
Peru	SPBLPGPT	06-Feb-2002	24-Jul-2019	912	3.47	−18.36	24.96	0.07	9.99
			(c) Forex – High Income Countries						
Canada	CAD	26-Aug-1998	24-Jul-2019	1092	1.20	−5.35	5.99	0.19	5.17
Denmark	DKK	17-Jan-1990	24-Jul-2019	1541	1.38	−10.19	7.26	−0.01	6.09
Norway	NOK	17-Jan-1990	24-Jul-2019	1541	1.57	−7.71	11.91	0.49	6.51
Australia	AUD	31-Jan-1990	24-Jul-2019	1539	1.53	−7.00	16.96	1.08	13.89
Switzerland	CHF	31-Jan-1990	24-Jul-2019	1539	1.55	−16.89	8.43	−0.96	14.77
UK	GBP	24-Jan-1990	24-Jul-2019	1540	1.32	−4.97	9.94	0.92	8.22
Japan	JPY	17-Jan-1990	24-Jul-2019	1541	1.41	−11.77	7.04	−0.57	7.88
Europe	EUR	17-Jan-1990	24-Jul-2019	1541	1.37	−10.17	7.70	−0.04	6.25
			(d) Forex – Emerging Countries (Latam)						
Argentina	ARS	05-Mar-2014	24-Jul-2019	282	2.57	−5.26	27.98	5.39	51.82
Brazil	BRL	02-Jun-1999	24-Jul-2019	1052	2.41	−18.59	18.35	0.47	13.38
Chile	CLP	17-Jan-1990	24-Jul-2019	1541	1.25	−6.36	9.22	0.26	6.83
Colombia	COP	02-Sep-1992	24-Jul-2019	1404	1.49	−6.37	8.68	0.31	6.83
Mexico	MXN	08-May-1996	24-Jul-2019	1212	1.49	−7.99	11.48	0.68	8.82
Peru	PEN	24-May-1995	24-Jul-2019	1262	0.65	−4.99	4.69	0.20	13.72

Switzerland, and the UK. In Latam markets, all asymmetry coefficients are positive, indicating predominance of depreciation episodes. Argentina has the highest asymmetry value, while kurtosis is higher in Argentina, Brazil, and Peru.

Figs. 1 and 2 show the evolution of (HI and Latam) stock and Forex market returns, respectively. We can see typical stylized facts such as clusters, higher volatility during the 2008–2009 Global Financial crisis (GFC), asymmetries, and departures from Normality. All these characteristics are present in both markets across all countries.

3.2. Results

The following models have been estimated for each market across all countries: (i) a single-regime GARCH (1,1)-$\mathscr{N}$ (described in 2.1); (ii) the MS-GARCH(1,1)-$\mathscr{N}$ model proposed by AGK (described in 2.2); (iii) the ABBC MS-GARCH(1,1) and MS-GJR(1,1) models (mentioned in 2.3) using innovations $\mathscr{N}$, $\mathscr{S}$, $sk\mathscr{N}$, and $sk\mathscr{S}$; and (iv) a single-regime GARCH/GJR model (described in 2.4) using innovations $\mathscr{S}$, $sk\mathscr{N}$ and $sk\mathscr{S}$. In addition to the models mentioned in (iii), three additional scenarios are contemplated: (i) imposing restrictions $\alpha_{1,1} = \alpha_{1,2}$ and $\beta_{1,1} = \beta_{1,2}$; (ii) imposing the restriction that the parameter $\nu$is the same in both regimes ($\nu_1 = \nu_2$); and (iii) imposing the restriction that the bias parameter $\xi$is the same in both regimes ($\xi_1 = \xi_2$). This implies estimating 144 models for each market across all countries. Since we have 15 stock markets and 14 Forex markets, the total number of models to estimate is 4,437. The single-regime GARCH model (1,1)-$\mathscr{N}$ appears in the first row of Tables 2a-3b. The AGK MS-GARCH-$\mathscr{N}$ model appears in the third row, while the best model obtained from the ABBC approach is shown in the fourth row. Following 2.4 above, the best single-regime GARCH/GJR model appears in the second row.[15] The best models are selected using several criteria: value of the log-marginal likelihood, significance of the parameters, and evaluation of the smoothed curve of probabilities for the high-volatility regime associated with a correct identification of the main domestic and international events involving volatility stress.

[15] Other specifications, such as the one suggested by Gray (1996), have also been considered and estimated. However, given the poor performance in terms of the significance of the parameters and the values of the log-marginal likelihood, these results have been ruled out. An earlier version of this paper includes those estimates; see Ataurima Arellano et al. (2017), where an MS-mean model (change only in the mean) is also estimated.

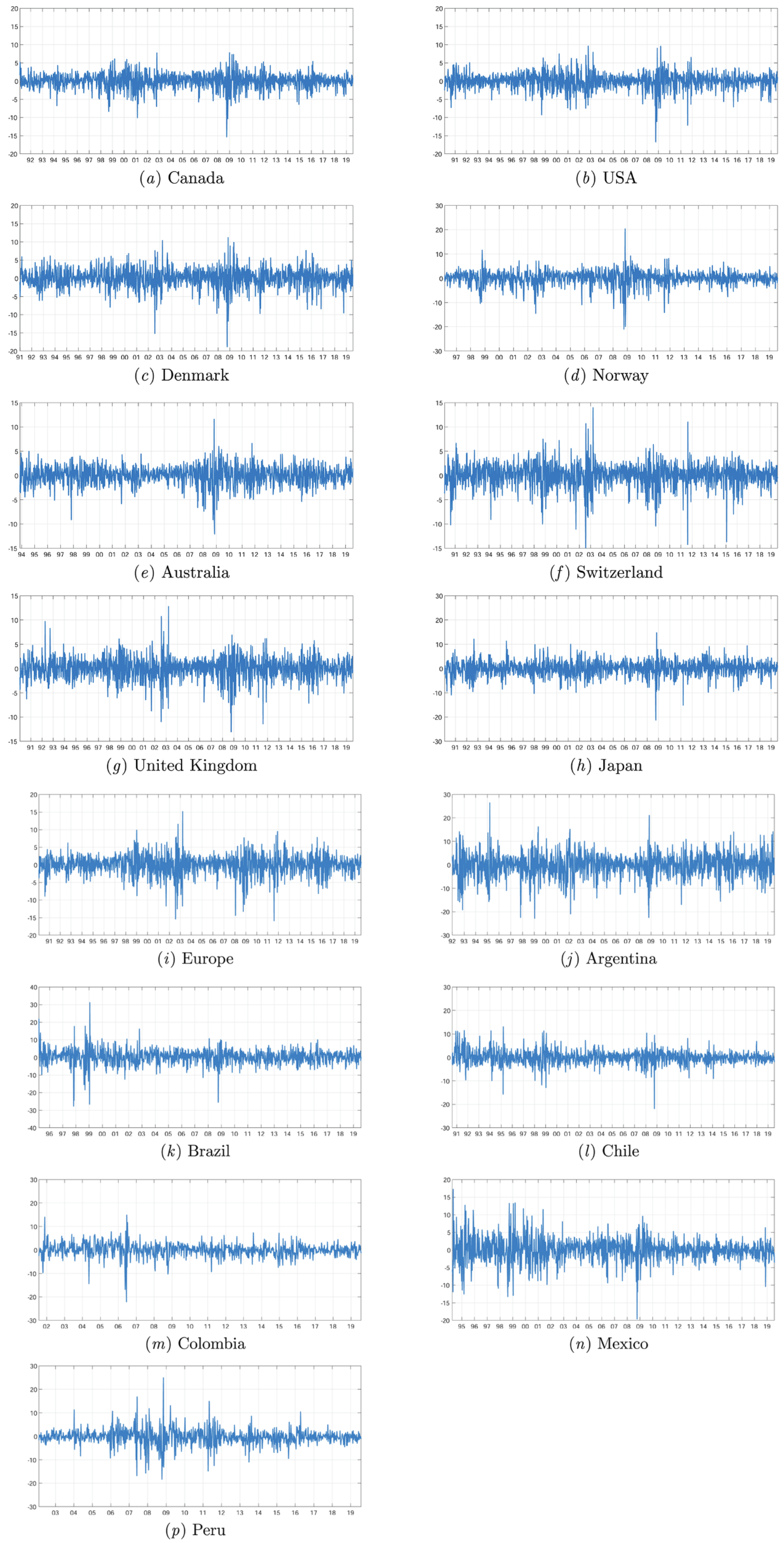


Fig. 1. Weekly High Income and Emerging (Latam) Stock Market Returns.

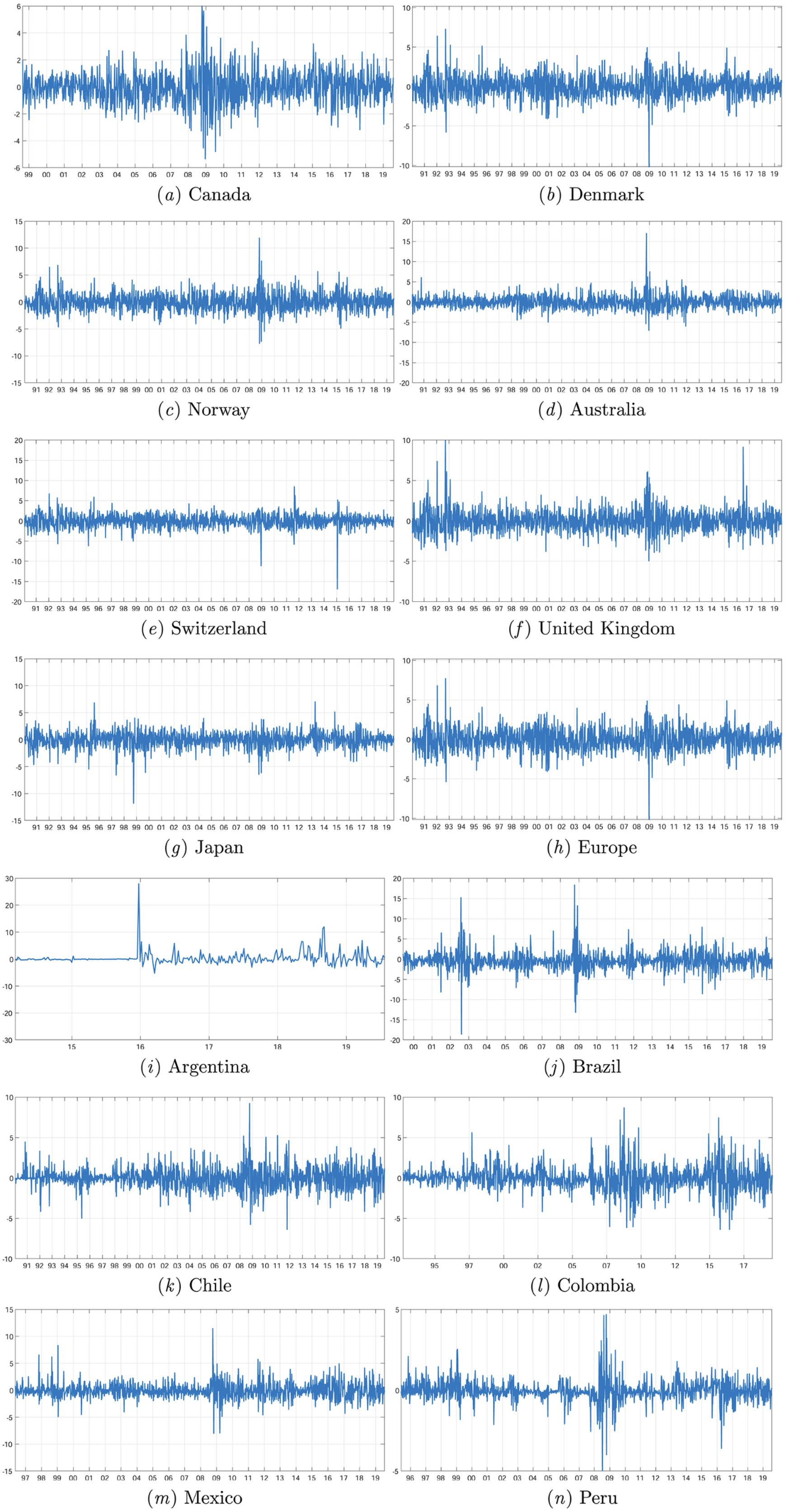


Fig. 2. Weekly High Income and Emerging (Latam) Forex Market Returns.

Table 2a
Estimated Parameters for Weekly High Income Stock Market Returns.

Model	$\alpha_{0,1}$	$\alpha_{1,1}$	$\alpha_{2,1}$	β_1	ν_1	χ_1	$\alpha_{0,2}$	$\alpha_{1,2}$	$\alpha_{2,2}$	β_2	ν_2	χ_2	p_{11}	p_{22}	log-lik
							Canada (SPTSX)								
GARCH-$\mathcal{N}$	0.164^a	0.147^a		0.817^a											−3019.011
GJR-sk$\mathcal{S}$	0.173^a	0.058^b	0.146^a	0.829^a	12.012^a	0.780^a									−2974.511
MS-GARCH-$\mathcal{N}$	0.318^c	0.083^b		0.745^a			1.712^a	0.083^b		0.745^a			0.981^a	0.952^a	−3013.149
MS-GARCH-$\mathcal{S}$	0.293^a	0.126^a		0.750^a	10.770^a		0.347^c	0.144^a		0.818^a	10.770^a		0.996^a	0.993^c	−3004.626
							USA (SPX)								
GARCH-$\mathcal{N}$	0.183^a	0.127^a		0.836^a											−3216.280
GJR-sk$\mathcal{S}$	0.176^a	0.006	0.203^a	0.844^a	7.791^a	0.742^a									−3105.362
MS-GARCH-$\mathcal{N}$	0.093^a	0.015		0.907^a			2.947^a	0.015		0.907^a			0.966^a	0.664^a	−3171.702
MS-GARCH-sk$\mathcal{N}$	0.238^a	0.105^a		0.788^a		0.674^a	0.917^a	0.105^a		0.788^a		0.833^a	0.990^a	0.982^c	−3152.032
							Denmark (KFX)								
GARCH-$\mathcal{N}$	0.331^a	0.116^b		0.838^a											−3444.548
GJR-sk$\mathcal{S}$	0.358^a	0.057^b	0.089^b	0.846^a	7.628^a	0.917^a									−3407.313
MS-GARCH-$\mathcal{N}$	0.049^b	0.008		0.921^a			2.151^a	0.008		0.921^a			0.925^a	0.698^a	−3425.307
MS-GJR-sk$\mathcal{N}$	0.099^b	0.038^a	0.064^b	0.867^a		0.974^a	0.951^a	0.038^a	0.115^b	0.867^a		0.736^a	0.729^a	0.359^a	−3403.474
							Norway (OSEBX)								
GARCH-$\mathcal{N}$	0.327^a	0.145^a		0.819^a											−2925.746
GJR-sk$\mathcal{S}$	0.323^a	0.012	0.121^a	0.877^a	10.443^a	0.741^a									−2866.553
MS-GARCH-$\mathcal{N}$	0.507^a	0.145^a		0.716^a			1.659^a	0.145^a		0.716^a			0.991^a	0.993^a	−2917.138
MS-GARCH-$\mathcal{S}$	0.146^a	0.045^a		0.916^a	21.261^c		2.921^a	0.045^a		0.916^a	21.261^c		0.985^a	0.817^a	−2897.624
							Australia (AS51)								
GARCH-$\mathcal{N}$	0.097^b	0.097^b		0.879^a											−2692.522
GJR-sk$\mathcal{S}$	0.139^a	0.000	0.171^a	0.873^a	76.181	0.809^a									−2656.686
MS-GARCH-$\mathcal{N}$	0.275^b	0.099^a		0.844^a			0.133^b	0.099^a		0.844^a			0.996^a	0.996^a	−2693.582
MS-GARCH-$\mathcal{N}$	0.055^b	0.040^c		0.937^a			4.886^c	0.319		0.266			0.994^a	0.949^a	−2685.804
							Switzerland (SMI)								
GARCH-$\mathcal{N}$	0.370^a	0.166^a		0.778^a											−3426.322
GJR-sk$\mathcal{S}$	0.513^a	0.034^c	0.215^a	0.761^a	8.583^a	0.758^a									−3340.298
MS-GARCH-$\mathcal{N}$	1.444^a	0.155^a		0.321^b			6.324^a	0.155^a		0.321^b			0.980^a	0.961^a	−3398.070
MS-GARCH-sk$\mathcal{N}$	1.137^a	0.174^a		0.442^a		0.745^a	3.976^a	0.174^a		0.442^a		0.748^a	0.985^a	0.975^a	−3367.963
							UK (UKX)								
GARCH-$\mathcal{N}$	0.245^a	0.148^a		0.807^a											−3289.774
GJR-sk$\mathcal{S}$	0.299^a	0.000	0.269^a	0.800^a	11.145^a	0.796^a									−3215.772
MS-GARCH-$\mathcal{N}$	0.156^a	0.020		0.888^a			2.306^a	0.020		0.888^a			0.963^a	0.768^a	−3270.870
MS-GARCH-sk$\mathcal{N}$	0.173^a	0.115^a		0.833^a		0.788^a	4.740^b	0.115^a		0.833^a		0.788^a	0.984^a	0.290^a	−3246.697
							Japan (NKY)								
GARCH-$\mathcal{N}$	1.138^a	0.127^a		0.753^a											−3850.347
GJR-sk$\mathcal{S}$	0.913^a	0.033^c	0.173^a	0.780^a	7.897^a	0.868^a									−3791.228
MS-GARCH-$\mathcal{N}$	0.741^a	0.004		0.772^a			5.739^a	0.004		0.772^a			0.925^a	0.799^a	−3821.982
MS-GARCH-sk$\mathcal{S}$	0.471^a	0.112^a		0.801^a	99.876^a	0.533^a	0.994^a	0.112^a		0.801^a	6.261^a	0.945^a	0.886^a	0.954^c	−3800.883
							Europe (SX5E)								
GARCH-$\mathcal{N}$	0.309^a	0.148^a		0.814^a											−3590.995
GJR-sk$\mathcal{S}$	0.320^a	0.054^a	0.146^a	0.824^a	10.118^a	0.755^a									−3518.341
MS-GARCH-$\mathcal{N}$	0.120^a	0.009		0.937^a			2.563^a	0.009		0.937^a			0.979^a	0.854^a	−3573.478
MS-GARCH-sk$\mathcal{N}$	0.293^a	0.050^b		0.859^a		0.756^a	1.505^a	0.183^a		0.704^a		0.756^a	0.993^a	0.991^b	−3537.931

a, b, c denote signicance level at 1%, 5% and 10% respectively.

3.2.1. Stock markets

Tables 2a (HI) and 2b (Latam) show the results for stock markets. A common result is that, in terms of the log-marginal likelihood, the data never validate the single-regime GARCH-$\mathcal{N}$ model. Furthermore, between the two alternative MS-GARCH models, ABBC obtain the highest value for the log-marginal likelihood (see the fourth row in both tables). Other results are as follows. First, the sk$\mathcal{N}$ distribution is selected for the U.S., Denmark, Switzerland, the UK, and Europe; the Student-t ($\mathcal{S}$) distribution is only chosen for Canada and Norway; the sk$\mathcal{S}$ distribution is validated by data for Japan; and Australia is the only case where the $\mathcal{N}$ innovation is selected. Second, in general the models require that the α_1 and β_1 parameters be different between regimes, except for the U.S., Denmark, Switzerland, the UK, and Japan, where the equality restriction ($\alpha_{1,1} = \alpha_{1,2}$ and $\beta_{1,1} = \beta_{1,2}$) is not rejected. Third, the parameter $\nu$is small, reflecting the presence and identification of heavy tails. The restriction that $\nu = 10.770$ is only used in Canada for both regimes, while Japan shows that regime 1 does not have heavy tails ($\nu = 99.876$), but regime 2 (high volatility) estimates that

Table 2b
Estimated Parameters for Weekly Emerging (Latam) Stock Market Returns

Model	$\alpha_{0,1}$	$\alpha_{1,1}$	$\alpha_{2,1}$	β_1	ν_1	χ_1	$\alpha_{0,2}$	$\alpha_{1,2}$	$\alpha_{2,2}$	β_2	ν_2	χ_2	p_{11}	p_{22}	log-lik
							Argentina (MERVAL)								
GARCH-$\mathcal{N}$	1.627^a	0.117^a		0.816^a											−4264.286
GJR-$sk\mathcal{S}$	0.961^a	0.066^b	0.089^a	0.848^a	7.737^a	0.879^a									−4222.621
MS-GARCH-$\mathcal{N}$	0.327^a	0.030^b		0.908^a			7.425^a	0.030^b		0.908^a			0.943^a	0.723^a	−4239.723
MS-GARCH-$sk\mathcal{N}$	0.101^b	0.016^a		0.964^a		0.876^a	3.957	0.075^b		0.866^a		0.876^a	0.960^a	0.914^a	−4224.661
							Brazil (IBOV)								
GARCH-$\mathcal{N}$	0.793^a	0.124^a		0.828^a											−3486.504
GJR-$sk\mathcal{S}$	1.203^a	0.052^b	0.134^a	0.811^a	8.757^a	0.815^a									−3438.180
MS-GARCH-$\mathcal{N}$	1.747^a	0.096^a		0.752^a			168.324^a	0.096^a		0.752^a			0.995^a	0.001	−3470.578
MS-GARCH-$sk\mathcal{S}$	1.155^a	0.122^a		0.817^a	11.479^c	1.091^a	1.021^a	0.122^a		0.817^a	7.787^a	0.721^a	0.996^a	0.991^c	−3441.919
							Chile (IPSA)								
GARCH-$\mathcal{N}$	0.103^a	0.100^c		0.892^a											−3526.751
GJR-$sk\mathcal{S}$	0.257^a	0.121^b	0.030	0.834^a	6.411^a	0.925^a									−3477.233
MS-GARCH-$\mathcal{N}$	0.168^a	0.000		0.915^a			3.116^a	0.000		0.915^a			0.965^a	0.806^a	−3490.151
MS-GARCH-$sk\mathcal{S}$	0.887^b	0.188^a		0.452^b	99.703^a	0.915^a	0.726^a	0.111^a		0.819^a	5.759^a	0.915^a	0.994^a	0.997^c	−3468.753
							Colombia (IGBC)								
GARCH-$\mathcal{N}$	0.552^b	0.149^b		0.787^a											−2236.533
GARCH-$sk\mathcal{S}$	0.879^a	0.193^b		0.702^a	5.504^a	0.911^a									−2196.250
MS-GARCH-$\mathcal{N}$	0.589^a	0.045^c		0.766^a			9.947^a	0.045^c		0.766^a			0.958^a	0.598^a	−2207.241
MS-GARCH-$sk\mathcal{S}$	0.014	0.006^c		0.979^a	11.155^b	0.864^a	0.484^b	0.124^b		0.871^a	11.155^b	0.864^a	0.894^a	0.898^a	−2191.718
							Mexico (MEXBOL)								
GARCH-$\mathcal{N}$	0.110^a	0.124^c		0.874^a											−3239.792
GJR-$sk\mathcal{S}$	0.130^a	0.070^a	0.110^a	0.868^a	8.824^a	0.877^a									−3206.752
MS-GARCH-$\mathcal{N}$	0.314^a	0.082^a		0.842^a			3.012^a	0.082^a		0.842^a			0.984^a	0.933^a	−3240.842
MS-GARCH-$\mathcal{N}$	0.068^b	0.039^b		0.941^a			1.753^b	0.039^b		0.941^a			0.984^a	0.814^b	−3221.202
							Peru (SPBLPGPT)								
GARCH-$\mathcal{N}$	0.353^a	0.177^a		0.796^a											−2257.710
GJR-$sk\mathcal{S}$	0.266^b	0.101^b	0.050	0.846^a	5.416^a	1.016^a									−2224.389
MS-GARCH-$\mathcal{N}$	0.202^a	0.000		0.922^a			7.828^a	0.000		0.922^a			0.979^a	0.797^a	−2227.305
MS-GARCH-$sk\mathcal{S}$	0.051^b	0.011^c		0.969^a	11.771^b	1.014^a	2.531	0.154^b		0.814^a	11.771^b	0.988^a	0.976^a	0.920^b	−2217.607

a, b, c denote signicance level at 1%, 5% and 10% respectively.

$\nu = 6.261$. Fourth, regarding the bias parameter (ξ), all estimates show values smaller than the unity, indicating a left bias; i.e., towards negative returns. This is consistent with the rational behavior hypothesis proposed by Samuelson (1970) and Lorenzo-Valdes and Ruiz-Porras (2014). Fifth, only Denmark shows leverage effects (MS-GJR), with $\alpha_{2,2} = 0.115$ for regime 2.

The calculation of unconditional volatilities in each regime suggests that regime 2 (high volatility) is usually twice as volatile as regime 1. There are cases where regime 2 is much more volatile relative to regime 1, like in Denmark (4 times), and Norway and the UK (5 times). The persistence of volatility in both regimes is also calculated using the best model estimated by ABBC. All countries, except Switzerland, show high persistence for the high-volatility regime (0.91 or higher). However, Switzerland shows the lowest persistence (0.616). Using the estimates performed by AGK, Denmark and Europe show the highest persistence, while the lowest persistence is observed in Switzerland (0.476) and Japan (0.773).

Table 2b (Latam) suggests the following. First, among ABBC models, the $\mathcal{N}$ distribution is only selected in the case of Mexico, whereas the $sk\mathcal{N}$ distribution is selected for Argentina. The $sk\mathcal{S}$ distribution is selected for the remaining Latam countries (Brazil, Chile, Colombia, and Peru), indicating the presence of bias and heavy tails in the distributions. Second, in general, the models require that the α_1 and β_1 parameters be different between regimes, except for Brazil and Mexico, where the equality restriction ($\alpha_{1,1} = \alpha_{1,2}$ and $\beta_{1,1} = \beta_{1,2}$) is not rejected. Third, the parameter ν(only estimated for Brazil, Chile, Colombia, and Peru) is small, reflecting the presence and identification of heavy tails. However, this parameter is fixed (around 11) in both regimes for Colombia and Peru. In the case of Chile, a large asymmetry is observed in the estimation of this parameter (99.703 in regime 1 and 5.759 in regime 2). This shows that regime 2 (high volatility) is characterized by heavy tails, which also happens in the case of Japan. Fourth, regarding the bias parameter (ξ), the estimates show values smaller than the unity, indicating a left bias; i.e., towards negative returns. This happens for all countries in the high-volatility region (regime 2). In the cases of Argentina, Chile, and Colombia, the hypothesis that this parameter is the same in both regimes ($\xi_1 = \xi_2$) cannot be rejected. Fifth, leverage effects (through MS-GJR) are not detected in any country.

The calculation of unconditional volatilities in each regime suggests that regime 2 (high volatility) is usually more volatile than regime 1. For example, in Chile the high-volatility regime is twice as volatile as regime 1, and considerably more in Argentina (4 times), Mexico (5 times), Peru (6 times), and Colombia (10 times). Only in Brazil are unconditional volatilities similar in both regimes. This finding is consistent with the findings shown in Table 2a (HI markets), because volatility in Latam countries is much

Table 3a
Estimated Parameters for Weekly High Income Forex Market Returns.

Model	$\alpha_{0,1}$	$\alpha_{1,1}$	$\alpha_{2,1}$	β_1	ν_1	χ_1	$\alpha_{0,2}$	$\alpha_{1,2}$	$\alpha_{2,2}$	β_2	ν_2	χ_2	p_{11}	p_{22}	log-lik
							Canada (CAD)								
GARCH-$\mathcal{N}$	0.016^{b}	0.068^{c}		0.919^{a}											−1641.517
GARCH-$sk\mathcal{S}$	0.015^{b}	0.067^{c}		0.921^{a}	97.589^{b}	1.043^{a}									−1641.028
MS-GARCH-$\mathcal{N}$	0.023^{b}	0.057^{a}		0.920^{a}			2.755^{c}	0.057^{a}		0.920^{a}			0.999^{a}	0.812^{a}	−1641.746
MS-GJR-$sk\mathcal{N}$	0.007^{a}	0.030^{a}	0.000^{a}	0.960^{a}		0.864^{a}	0.074^{a}	0.135^{a}	0.000^{a}	0.841^{a}		1.415^{a}	0.984^{a}	0.961^{a}	−1633.873
							Denmark (DKK)								
GARCH-$\mathcal{N}$	0.033^{a}	0.062^{b}		0.920^{a}											−2605.826
GARCH-$sk\mathcal{S}$	0.037^{a}	0.059^{b}		0.920^{a}	10.632^{a}	1.031^{a}									−2589.642
MS-GARCH-$\mathcal{N}$	0.021^{b}	0.000		0.970^{a}			0.250^{a}	0.000		0.970^{a}			0.979^{a}	0.886^{a}	−2594.190
MS-GARCH-$sk\mathcal{S}$	0.006^{c}	0.017^{b}		0.975^{a}	11.570^{a}	1.020^{a}	0.149^{c}	0.075^{b}		0.877^{a}	11.570^{a}	1.020^{a}	0.991^{a}	0.980^{c}	−2585.717
							Norway (NOK)								
GARCH-$\mathcal{N}$	0.117^{a}	0.093^{a}		0.858^{a}											−2788.855
GARCH-$sk\mathcal{S}$	0.118^{a}	0.085^{a}		0.863^{a}	14.729^{a}	1.120^{a}									−2774.963
MS-GARCH-$\mathcal{N}$	0.064^{b}	0.069^{a}		0.861^{a}			0.253^{a}	0.069^{a}		0.861^{a}			0.919^{a}	0.932^{a}	−2787.339
MS-GARCH-$sk\mathcal{S}$	0.005	0.037^{b}		0.954^{a}	18.784^{b}	0.815^{a}	0.179^{a}	0.092^{a}		0.847^{a}	18.784^{b}	1.319^{a}	0.975^{a}	0.984^{b}	−2768.436
							Australia (AUD)								
GARCH-$\mathcal{N}$	0.070^{a}	0.089^{b}		0.881^{a}											−2703.267
GARCH-$sk\mathcal{S}$	0.045^{a}	0.067^{b}		0.913^{a}	11.457^{a}	1.151^{a}									−2680.467
MS-GARCH-$\mathcal{N}$	0.070^{a}	0.020^{b}		0.923^{a}			1.075^{a}	0.020^{b}		0.923^{a}			0.989^{a}	0.828^{a}	−2690.934
MS-GJR-$sk\mathcal{S}$	0.038^{a}	0.038^{a}	0.000^{a}	0.936^{a}	78.338^{a}	1.138^{a}	0.877^{a}	0.000^{a}	0.000^{a}	0.901^{a}	6.932^{a}	1.101^{a}	0.994^{a}	0.924^{a}	−2679.270
							Switzerland (CHF)								
GARCH-$\mathcal{N}$	0.216^{a}	0.082^{a}		0.833^{a}											−2818.165
GJR-$sk\mathcal{S}$	0.057^{b}	0.051^{c}	0.008	0.919^{a}	7.004^{a}	0.914^{a}									−2721.105
MS-GARCH-$\mathcal{N}$	0.043^{a}	0.000		0.939^{a}			1.338^{a}	0.000		0.939^{a}			0.960^{a}	0.489^{a}	−2736.796
MS-GARCH-$sk\mathcal{S}$	0.007	0.008^{c}		0.978^{a}	7.714^{a}	0.916^{a}	0.268^{b}	0.041^{a}		0.854^{a}	7.714^{a}	0.916^{a}	0.990^{a}	0.997^{c}	−2710.422
							UK (GBP)								
GARCH-$\mathcal{N}$	0.044^{b}	0.073^{c}		0.902^{a}											−2498.914
GARCH-$sk\mathcal{S}$	0.045^{a}	0.057^{b}		0.915^{a}	9.400^{a}	1.111^{a}									−2463.837
MS-GARCH-$\mathcal{N}$	0.315^{a}	0.020		0.918^{a}			0.067^{a}	0.020		0.918^{a}			0.998^{a}	0.984^{a}	−2477.169
MS-GJR-$sk\mathcal{S}$	0.045^{a}	0.018^{a}	0.000^{a}	0.940^{a}	33.968^{a}	1.020^{a}	0.778^{a}	0.118^{a}	0.003^{a}	0.664^{a}	8.830^{a}	1.322^{a}	0.996^{a}	0.981^{a}	−2453.528
							Japan (JPY)								
GARCH-$\mathcal{N}$	0.098^{a}	0.090^{a}		0.864^{a}											−2662.570
GJR-$sk\mathcal{S}$	0.085^{a}	0.058^{b}	0.017	0.889^{a}	6.424^{a}	0.908^{a}									−2605.315
MS-GARCH-$\mathcal{N}$	0.061^{a}	0.000		0.913^{a}			0.699^{a}	0.000		0.913^{a}			0.952^{a}	0.768^{a}	−2621.528
MS-GARCH-$sk\mathcal{S}$	0.001	0.006^{b}		0.988^{a}	8.013^{a}	0.949^{a}	0.154^{b}	0.041^{b}		0.905^{a}	8.013^{a}	0.864^{a}	0.960^{a}	0.968^{b}	−2602.260
							Europe (EUR)								
GARCH-$\mathcal{N}$	0.035^{a}	0.066^{b}		0.916^{a}											−2586.209
GARCH-$sk\mathcal{S}$	0.037^{a}	0.062^{b}		0.917^{a}	10.526^{a}	1.014^{a}									−2570.000
MS-GARCH-$\mathcal{N}$	0.025	0.000		0.963^{a}			0.474^{a}	0.000		0.963^{a}			0.975^{a}	0.755^{a}	−2572.376
MS-GARCH-$\mathcal{S}$	0.009^{c}	0.019^{b}		0.970^{a}	11.419^{a}		0.150^{c}	0.078^{a}		0.871^{a}	11.419^{a}		0.989^{a}	0.981^{c}	−2566.737

a, *b*, *c* denote signicance level at 1%, 5% and 10% respectively.

higher in regime 2 (high volatility). The persistence of volatility in both regimes is also higher in these markets compared to Table 2a (HI markets). The most persistent are Colombia (0.995), Mexico (0.980), and Peru (0.968), while the least persistent are Brazil (0.939) and Chile (0.930). These values indicate that the high-volatility regime (regime 2) in Latam markets is more persistent than in HI markets. This implies that these episodes of turbulence are longer and more uncertain in Latam markets and that the adjustment is usually slow. Using AGK estimates, the persistence values of the high-volatility regime are usually lower than the ones indicated above, but remain higher compared to those of HI markets.

3.2.2. Forex markets

Tables 3a (HI) and 3b (Latam) show the results for Forex markets. As in Tables 2a-2b, in terms of log-marginal likelihood, the single-regime GARCH-$\mathcal{N}$ model is never validated by the data, except for Canada, where a similar fit as in AGK is observed. The models estimated using ABBC yield the highest log-marginal likelihood value (see the fourth row). Other results are as follows. First, among the models estimated by ABBC, the $\mathcal{N}$ distribution is never selected, which is an important difference with the results for stock markets. The $sk\mathcal{N}$ distribution is selected for Canada and the $\mathcal{S}$ for Europe. For most remaining countries (six), $sk\mathcal{S}$ disturbances are selected, indicating the presence of bias and heavy tails in the distributions, in contrast with stock markets (Table 2a). Second, all countries require different α_1 and β_1 between regimes; i.e., unlike in some stock markets, the hypothesis that $\alpha_{1,1} = \alpha_{1,2}$ and

Table 3b
Estimated Parameters for Weekly Emerging (Latam) Forex Market Returns.

Model	$\alpha_{0,1}$	$\alpha_{1,1}$	$\alpha_{2,1}$	β_1	ν_1	χ_1	$\alpha_{0,2}$	$\alpha_{1,2}$	$\alpha_{2,2}$	β_2	ν_2	χ_2	p_{11}	p_{22}	log-lik
							Argentina (ARS)								
GARCH-$\mathcal{N}$	4.199^a	0.147^c		0.254^c											−658.423
GARCH-sk$\mathcal{S}$	0.000^c	0.033		0.967^a	2.467^a	1.356^a									−391.582
MS-GARCH-$\mathcal{N}$	0.002	0.798^a		0.416^a			44.795^a	0.798^a		0.416^a			0.967^a	0.268	−420.537
MS-GARCH-sk$\mathcal{S}$	0.000^a	0.065^a		0.934^a	2.187^a	1.512^a	2.732^a	0.221^a		0.779^a	2.187^a	1.268^a	0.986^a	0.988^a	−379.335
							Brazil (BRL)								
GARCH-$\mathcal{N}$	0.245^a	0.173^a		0.791^a											−2270.085
GARCH-sk$\mathcal{S}$	0.301^a	0.175^b		0.798^a	5.414^a	1.242^a									−2223.117
MS-GARCH-$\mathcal{N}$	0.183^a	0.064^b		0.841^a			7.056^a	0.064^b		0.841^a			0.974^a	0.505^a	−2241.482
MS-GARCH-sk$\mathcal{S}$	0.639^b	0.182^a		0.751^a	3.343^a	1.798^a	0.577^b	0.182^a		0.751^a	6.354^a	1.138^a	0.989^a	0.966^b	−2220.142
							Chile (CLP)								
GARCH-$\mathcal{N}$	0.036^a	0.115^a		0.863^a											−2383.337
GJR-sk$\mathcal{S}$	0.000	0.071	0.000	0.928^a	4.914^a	1.041^a									−2294.421
MS-GARCH-$\mathcal{N}$	0.008^a	0.049^a		0.744^a			0.607^a	0.049^a		0.744^a			0.847^a	0.854^a	−2308.896
MS-GARCH-sk$\mathcal{S}$	0.000	0.003^c		0.997^a	2.143^a	1.121^a	0.042^b	0.086^a		0.898^a	7.054^a	1.121^a	0.968^a	0.994^b	−2226.545
							Colombia (COP)								
GARCH-$\mathcal{N}$	0.047^a	0.154^a		0.826^a											−2293.218
GARCH-sk$\mathcal{S}$	0.018^b	0.119^b		0.871^a	5.734^a	1.095^a									−2232.589
MS-GARCH-$\mathcal{N}$	0.002	0.011		0.924^a			0.654^a	0.011		0.924^a			0.944^a	0.786^a	−2246.028
MS-GARCH-sk$\mathcal{S}$	0.003	0.085^a		0.902^a	5.785^a	1.094^a	0.165^b	0.085^a		0.902^a	21.396	1.267^a	0.944^a	0.757^b	−2218.962
							Mexico (MXN)								
GARCH-$\mathcal{N}$	0.113^a	0.099^a		0.851^a											−2112.664
GARCH-sk$\mathcal{S}$	0.067^a	0.121^b		0.854^a	5.473^a	1.270^a									−2014.534
MS-GARCH-$\mathcal{N}$	0.069^a	0.000		0.885^a			1.739^a	0.000		0.885^a			0.954^a	0.618^a	−2048.274
MS-GARCH-sk$\mathcal{S}$	0.114^a	0.102^a		0.826^a	3.875^a	1.307^a	0.226^b	0.102^a		0.826^a	9.470^a	1.226^a	0.994^a	0.993^c	−2008.449
							Peru (PEN)								
GARCH-$\mathcal{N}$	0.011^a	0.191^a		0.790^a											−979.722
GARCH-sk$\mathcal{S}$	0.015^a	0.229^b		0.757^a	4.104^a	1.192^a									−899.181
MS-GARCH-$\mathcal{N}$	0.003	0.084^a		0.800^a			0.196^a	0.084^a		0.800^a			0.913^a	0.658^a	−930.948
MS-GJR-sk$\mathcal{S}$	0.015^a	0.325^b	0.000^a	0.606^a	4.018^a	2.466^a	0.034^a	0.197^a	0.000^a	0.761^a	4.018^a	1.110^a	0.920^a	0.982^b	−880.529

a, *b*, *c* denote signicance level at 1%, 5% and 10% respectively.

$\beta_{1,1} = \beta_{1,2}$ is always rejected. Third, the parameter ν(estimated for all countries except Canada) shows values that may be considered suitable for capturing heavy tails. In Denmark and Europe, values of $\nu$are around 11. Japan and Switzerland show smaller estimates (between 7.714 and 8.013) and in both cases the hypothesis that this parameter is the same ($\nu_1 = \nu_2$) in both regimes is not rejected. Norway shows a fixed νestimate of 18.784 for both regimes. In the cases of the UK and Australia, an interesting asymmetry is observed in the estimates for ν. In regime 1, the value of this parameter is high and the existence of Normality can be suggested. However, in regime 2 (high volatility), $\nu = 8.830$ and $\nu = 6.932$ for the UK and Australia, respectively, indicating heavy tails in the high-volatility regime. Fourth, regarding the bias parameter (ξ), the estimates show values greater than unity for Canada, Norway, Australia, the UK, and Denmark. In this case, values greater than the unity indicate a bias towards the right side (positive returns values); i.e., a greater presence of depreciation episodes. The most notable cases are Canada, Norway, and the UK. In the cases of Switzerland and Japan, the values are lower than the unity, suggesting a bias towards the appreciation of domestic currencies. Only in Switzerland and Denmark the hypothesis that this parameter is the same ($\xi_1 = \xi_2$) in both regimes cannot be rejected. Fifth, positive leverage effects (MS-GJR) are observed in Canada, Australia, and the UK, but the magnitudes are extremely small.

The calculation of unconditional volatilities in each regime suggests that regime 2 is usually more volatile than regime 1. In most cases (Canada, Denmark, Norway, Switzerland, The UK, and Europe), regime 2 is twice as volatile as regime 1 (4 and 2.5 times higher in Japan and Australia, respectively). In general, volatility is more moderate than in HI stock markets (Table 2a). The persistence of volatility in both regimes is high. The most persistent case is Canada (0.979), with a value around 0.95 in the other countries. The lowest levels of persistence occur in Switzerland (0.985), Australia (0.902), and the UK (0.784). Using AGK estimates, the most persistent countries are Canada, and Europe, while the rest show lower levels. However, compared with the stock market (Table 2a), AGK does not seem to underestimate the probability p_{22}.

Like previous Tables, Table 3b shows that, in terms of the log-marginal likelihood values, the single-regime GARCH-$\mathcal{N}$ model is never validated by the data, Argentina being the clearest example. The models estimated using ABBC yield the highest log-marginal likelihood values (fourth row). Other comments are as follows. First, among the different models estimated using ABBC, the $\mathcal{N}$, sk$\mathcal{N}$ and *S* distributions are never selected, which is an important difference with Latam stock markets (Table 2b). In all cases, sk$\mathcal{S}$ disturbances are chosen, indicating the presence of bias and heavy tails in the distributions. The sk$\mathcal{S}$ distribution was selected for most HI Forex markets, but in Latam countries the evidence is stronger. Second, half of the countries (Brazil, Colombia, and Mexico) do not reject the hypothesis that $\alpha_{1,1} = \alpha_{1,2}$ and $\beta_{1,1} = \beta_{1,2}$, while for the other half (Argentina, Chile, and Peru) the evidence points to the opposite. Third, the results for the parameter $\nu$are clearly different from those obtained for stock markets (HI and Latam, Tables 2a and 2b) and HI Forex markets (Table 3a), because the values are smaller; e.g., $\nu = 2.187$ in Argentina for both regimes. In the other countries the values are between 2.143 and 9.470, thereby rejecting the hypothesis that $\nu_1 = \nu_2$ in most countries. Only Colombia shows $\nu = 21.396$ in regime 2 (high volatility) compared to $\nu = 5.785$ for regime 1. Fourth, regarding the bias parameter (ξ), the estimates show values greater than unity in all countries and for both regimes. In addition, the values have magnitudes greater than those found in all other markets and countries. Values greater than the unity indicate a bias towards the right; i.e., greater presence of depreciation episodes. The most notable cases are Argentina, Colombia, and Mexico. Fifth, leverage effects (MS-GJR) are only observed for Peru, the magnitude being very small in regime 1, while in regime 2 the value is 0.034, higher than for HI markets.

The calculation of unconditional volatilities in each regime suggests that regime 2 (high volatility) is usually more volatile than regime 1. The lowest levels are for Mexico and Peru, where volatility in regime 2 is 1.5–2.0 times the volatility in regime 1. However, in the other countries, volatility in regime 2 is much higher than in regime 1; i.e., 8, 20, and 60 times greater in Colombia, Chile, and Argentina (the most extreme case), respectively. The values suggest a more volatile behavior in regime 2 compared with HI Forex markets and Latam stock markets. In the case of Peru (one of the lowest), it seems that central bank intervention in the Forex market reduces stress in the high-volatility regime. The persistence of volatility in both regimes is high: Argentina 0.999, Chile 0.983, Colombia 0.987, and Peru 0.957. The other countries show less persistent high-volatility regimes (0.928 or 0.933). Using AGK estimates, Argentina shows an explosive persistence (1.210), given that covariance stationarity is not imposed. The least persistent are Chile (0.793), Peru (0.883), and Mexico (0.893). We confirm that AGK always estimates lower persistence than ABBC, except in the case of Argentina.

3.2.3. *Identification and characteristics of high-volatility periods*

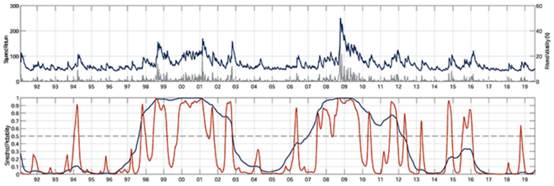
Figs. 3 and 4 show two panels for each country. The upper panel shows two series: squared returns (gray) and filtered volatility (blue) extracted from the best-estimated model using ABBC; see fourth row in Tables 3a through 3b. The lower panel shows two series of smoothed probabilities associated with regime 2 (high volatility): those extracted using AGK in red and those extracted using the ABBC approach in blue. Both panels show how well the smoothed probabilities follow the patterns of the filtered volatilities, which in turn follow the behavior of the squared returns. Figs. 3 and 4 show the results for the stock and Forex markets, respectively.

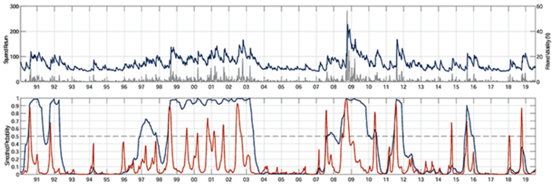
Four aspects are evidenced from Figs. 3 and 4. First, in most of cases, the smoothed probabilities obtained from the best models estimated using ABBC reflect better the behavior of the filtered volatilities. This is achieved by including skewed innovations such as $sk\mathcal{N}$ and $sk\mathcal{S}$. In various cases, the smoothed probabilities obtained from the ABBC models seem to form an envelope of the probabilities extracted by AGK. In general, the distributional flexibility allowed by ABBC produces more persistent smoothed probabilities in regime 2. This flexibility is not allowed by AGK. Second, although the log-marginal likelihoods of AGK are smaller than those obtained by ABBC, there are some cases where the smoothed probabilities are better behaved under the first approach; i.e., the smoothed probabilities are more similar to the movements of the filtered volatilities (AGK envelopes ABBC). Even comparing with the evolution of the returns (Figs. 1 and 2) we find the same observation. Third, there are cases where both types of methods allow noise and seem to try to capture all movements of filtered volatilities. Fourth, in some cases it is difficult to argue in favor of one of the two methods or models, given the noise in both series of smoothed probabilities under regime 2.

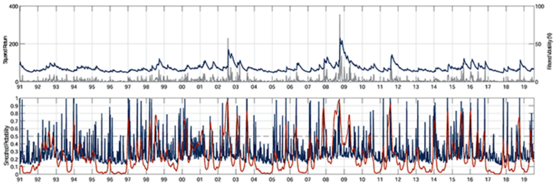
Fig. 3 shows the case of HI and Latam stock markets. The smoothed probabilities of regime 2 are best identified (not noisy) for Canada, the U.S., and Europe when ABBC is used. The blue line looks like an envelope of the red line (AGK). However, it is worth noting the following aspects: (i) the blue line appears as the envelope for 1997–2003 and 2007–2012, covering the GFC (2008–2009) and the European debt crisis; and (ii) the small jumps on the blue line (upper panel) in 1994 are only captured by AGK. The same happens with the 2015–2016 and 2019 episodes. In sum, ABBC MS-GARCH models capture volatility persistence in regime 2, but fail to capture some particular jumps. Similar results are observed for the U.S. market, while the estimation made using ABBC seems to capture well the probabilities of regime 2. The case of Europe is another good illustration of the qualities of the ABBC method compared with AGK. However, an opposite result occurs for Norway and Australia. In both cases, the probabilities of regime 2 estimated by AGK are more persistent and appear to be an envelope of the results obtained from ABBC, although in the case of Australia there seems to be an overestimation of the probabilities under regime 2 using AGK. In the case of Switzerland, both models capture in the same way (almost identical) the probabilities and persistence of regime 2. In the cases of Denmark, UK, and Japan it is difficult to decide which of the two models performs better. In Denmark, both estimates of the smoothed probabilities are very noisy. However, the red line (AGK) seems to perform relatively better. A similar observation can be made in the case of the UK. The case of Japan is the most difficult to pin down.

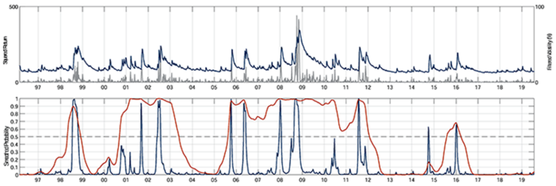
Fig. 3 also shows the results for Latam stock markets. In the cases of Brazil and Chile, the smoothed probabilities for the high-volatility regime are well captured by ABBC; i.e., while AGK identifies four points associated with the high-volatility regime, the ABBC approach considers wide and persistent probabilities. In the case of Mexico, the smoothed probabilities obtained using AGK appear as the envelope of the other method. However, filtered volatilities (top panel) suggest jumps in 2011–2012 and 2019, which are only detected by ABBC (blue line). In the case of Peru, similar smoothed probabilities are observed, but the ABBC approach offers better results. In the cases of Argentina and Colombia, noisy probabilities are obtained, although with relative advantage for the specification estimated using ABBC.

Fig. 4 shows the case of HI and Latam Forex markets. The first Figures correspond to HI markets. Unlike stock markets, ABBC show more persistence in regime 2 (high volatility) than AGK. In Canada, Switzerland, and Europe this seems to be clearly the case. In the cases of Denmark, Australia, and the UK, both approaches yield similar results in terms of the smoothed probabilities of regime 2. Norway appears as a difficult case. Initially both methods work in a similar way, but ABBC offer better results starting in 1999.


(a) Canada


(b) USA


(c) Denmark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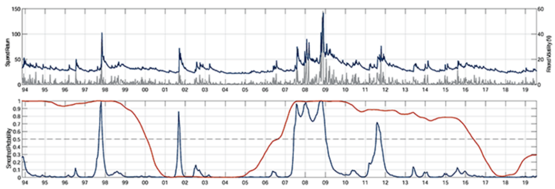
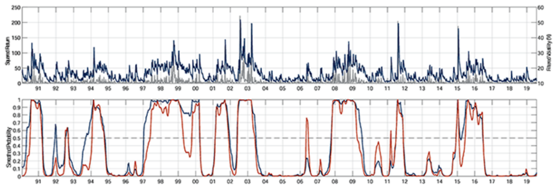

(e) Australia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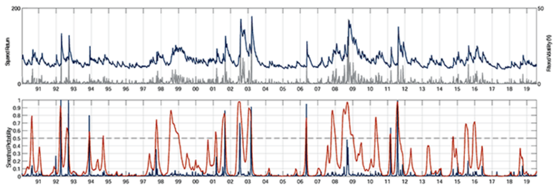
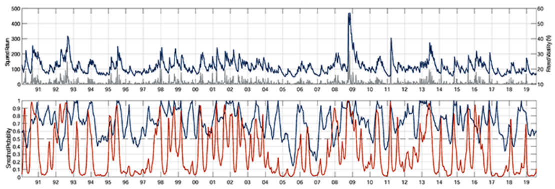
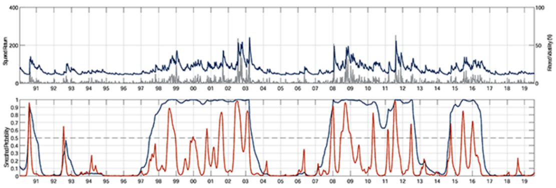
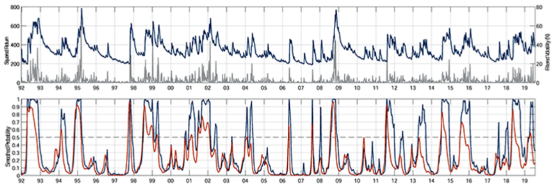
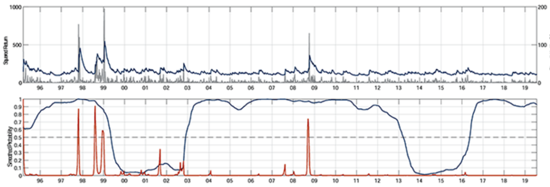
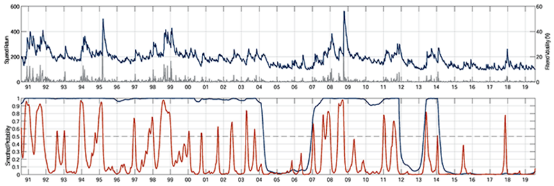

(g) United Kingdom

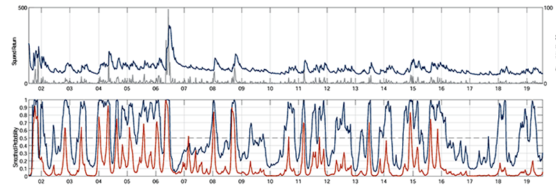

(h) Japan

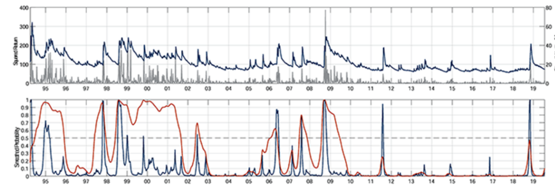

(i) Europe

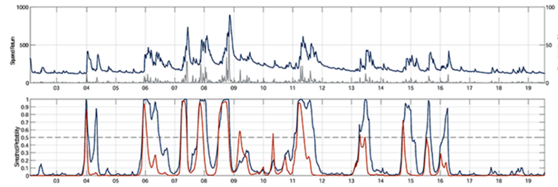

(j) Argentina


(k) Brazil


(l) Chile


(m) Colombia


(n) Mexico


(p) Peru

(caption on next page)

Fig. 3. Filtered Volatility and Smoothed Probabilities for Stock Markets Returns. First panels shows filtered volatility using the MSGARCH model of ABBC (blue line) and the square of the returns (gray line). Second panels shows: smoothed probabilities of high volatility regime of MSGARCH model of AGK (red line); and the best MSGARCH model of ABBC (blue line).

The rest of Fig. 4 shows the case of Latam Forex markets. Argentina is a special case, as long as it maintained a fixed exchange rate regime for a long time. Indeed, Argentina shows the highest filtered volatility values across all countries in both markets. AGK selects only a small number of weeks as belonging to the high-volatility regime. However, ABBC identify a small jump in 2015, and all observations from 2016 until the end of the sample are cataloged as regime 2 observations. In the case of Brazil, ABBC estimates seem like an envelope of the results obtained using the other method. Chile also shows a very persistent high-volatility regime. AGK seems to perform relatively better in the case of Colombia. In the case of Mexico, both methods yield similar estimates, but ABBC capture additional high-volatility observations in 2011–2012 and 2019. In the case of Peru, both methods yield similar performances, although ABBC offer relatively better estimates.

From the previous analysis, we can distinguish three main stress episodes which are common across countries and markets over time: (i) 1994–2004 (Dotcom bubble, September 11 attacks); (ii) 2007–2012 (subprime mortgage crisis, GFC, European debt crisis); and (iii) 2014–2016 (China's economic slowdown, trade wars, oil price crisis). First, we count the number of countries experiencing high-volatility episodes per week. A main observation is that ABBC capture more countries facing high volatility than AGK, which is more notorious in Latam Forex markets. For instance, AGK does not identify any Latam stock market in the third stress period, but does for Forex markets in the first stress period.

Second, for simplification, subsets of countries are chosen for each period of common stress, where some of the MS-GARCH methods capture at least one high-volatility period.[16] For explanatory purposes, we take the Canadian stock market for the first stress period. In this case, AGK detects 5 high-volatility sub-periods with different duration (172 weeks in all). However, using an MS-GARCH-$\mathscr{S}$, ABBC select only one long high-volatility period lasting 271 weeks. A similiar issue is observed for the second stress period. Other very similar examples are the U.S. and Chile. In the case of the U.S., AGK detects 7 high-volatility sub-periods (48 weeks in all). At the same time, using an MS-GARCH-$sk\mathscr{N}$, ABBC select only two long sub-periods (47 and 257 weeks, respectively). In the case of Chile, AGK identifies 12 high-volatility sub-periods (111 weeks in all), whereas ABBC consider only one very long 713-week sub-period (using an MS-GARCH-$sk\mathscr{S}$). The results are similar for the second and third stress periods in Europe, Peru, and Colombia. In all these cases, ABBC look like an envelope of AGK. A special case is Mexico, where a similar specification (MS-GARCH-$\mathscr{N}$ with $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$) was estimated by AGK and ABBC, but the persistence of the-high volatility regime of the former envelopes the latter.

Findings are similar in Forex markets: a relevant case is Canada, where AGK is unable to detect any high-volatility period in the first and third stress periods, while ABBC do it using an MS-GJR-$sk\mathscr{N}$.

3.2.4. Other models

While ABBC are better than AGK models in measuring performance in terms of the log-marginal likelihood, there are other single-regime models that may yield better log-marginal likelihood values. These are simpler models in that they do not require non-linear MS-type modeling, as mentioned in 2.4. These are single-regime GARCH or GJR models, but with flexibility in the distribution, like Ardia et al. (2019a) and Ardia et al. (2019b). We have selected the best models according to the values of the log-marginal likelihood (second row of Tables 2 and 3). Many of these models are not only better than the single-regime GARCH-$\mathscr{N}$ model (first row of Tables 2 and 3), but in several cases they outperform the best models selected using ABBC.

In HI stock markets (Table 2a), all models, except for Denmark, have better log-marginal likelihood values compared with ABBC. Another important detail is that, in all cases, the selected model is a single-regime GJR-$sk\mathscr{S}$. Other points to highlight are the following. First, the parameter $\nu$is between 7.897 (Japan) and 12.612 (Canada). In all cases it is statistically significant, except for Australia, where a higher value is obtained ($\nu = 76.181$). Second, the bias parameter (ξ) is always smaller than unity, indicating asymmetries towards negative returns. Third, there are leverage effects in all cases, all statistically significant. The values are high and range between 0.121 (Norway) and 0.269 (UK). Fourth, the volatility persistence values are between 0.902 and 0.960.

In the case of Latam stock markets (Table 2b), only in Colombia, Chile, and Peru, ABBC modeling yields better log-marginal likelihood values. In the remaining cases (Argentina, Brazil, and Mexico), the single-regime GJR-$sk\mathscr{S}$ model (the same model as in the case of selected HI markets) a better fit (Table 2a). Thus, we note the following: (i) the values of parameter $\nu$are smaller than those for HI stock markets (Table 2a); (ii) the bias parameter (ξ) is also higher than for HI stock markets; and (iii) the leverage effects are smaller than for HI stock markets (and are not even significant for Chile).

In the case of the HI Forex markets (Table 3a), the results obtained from ABBC are hard to beat. Only in the case of Australia a single-regime GARCH-$sk\mathscr{S}$ model manages to have a log-marginal likelihood almost equal to that obtained using ABBC. In this case $\nu = 11.457$, while the asymmetry parameter $\xi = 1.151$. The latter reflects a bias towards the right side, i.e., a predominance of depreciation episodes. The persistence of the high-volatility regime is very similar to that obtained with ABBC. In the cases of Japan and Europe, the log-marginal likelihoods of these models are close to those obtained by ABBC, always using a $sk\mathscr{S}$ distribution. In sum, for this market there are advantages in using MS-GARCH/MS-GJR models, especially those selected using ABBC.

Table 3b shows the results for Latam Forex markets. In no case can a single-regime GARCH or GJR model beat an MS-GARCH/MS-GJR model estimated using ABBC. The only case where the log-marginal likelihoods are close is Brazil, where an $sk\mathscr{S}$ distribution is

[16] A detailed description for all countries is available upon request.

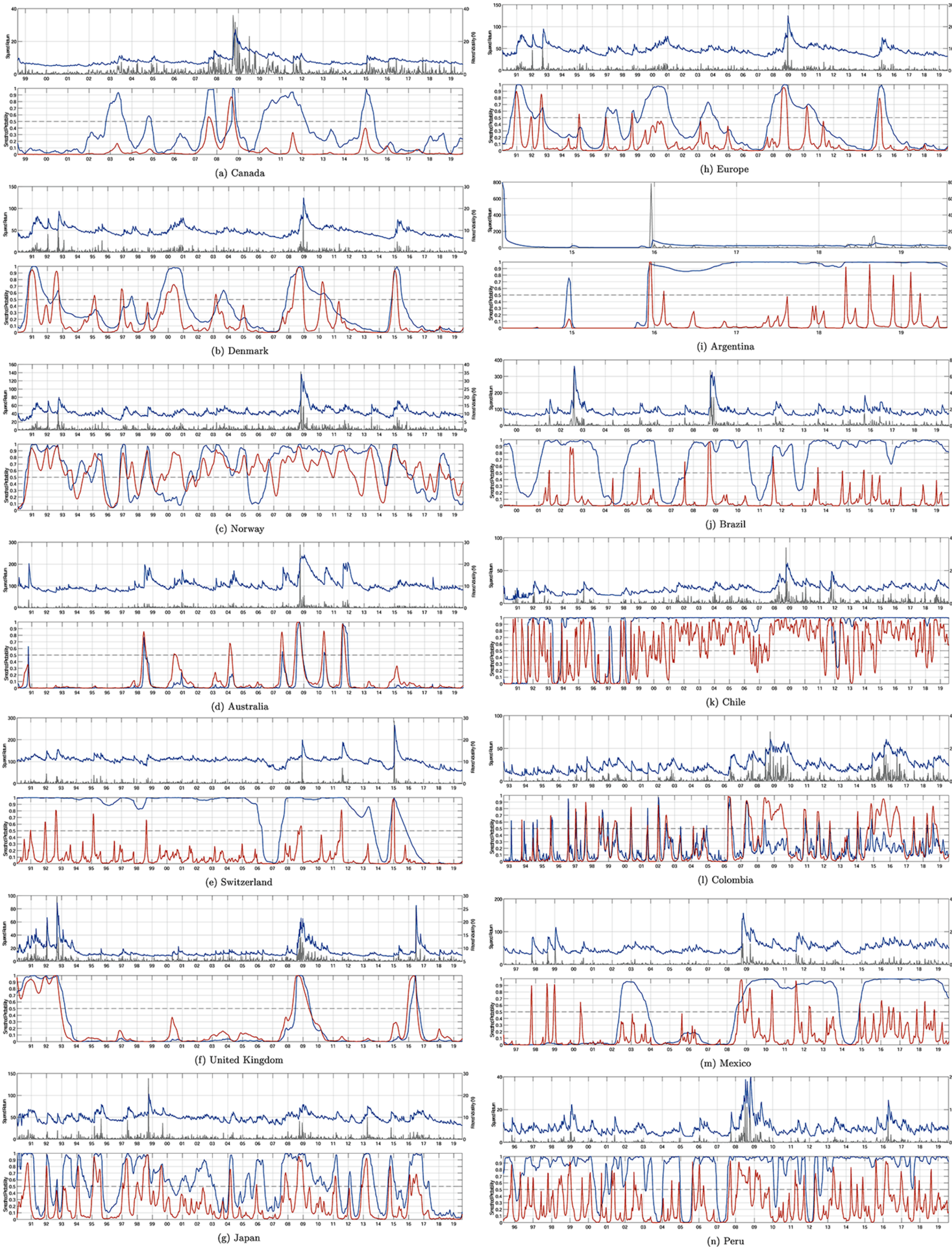


Fig. 4. Filtered Volatility and Smoothed Probabilities for Forex Markets Returns. First panels shows filtered volatility using the MSGARCH model of ABBC (blue line) and the square of the returns (gray line). Second panels shows: smoothed probabilities of high volatility regime of MSGARCH model of AGK (red line); and the best MSGARCH model of ABBC (blue line).

selected again.

The following are some conclusions about this discussion: (i) there are some single-regime models with flexible distributional characteristics that yield better log-marginal likelihood values; (ii) in (HI and Latam) stock markets, a single-regime model always yields a better log-marginal likelihood than ABBC (with the only exception of Chile); (iii) regarding (HI and Latam) Forex markets, there is a larger number of countries where the log-marginal likelihood of ABBC MS-GARCH/MS-GJR models is higher; i.e., compared with stock markets, in Forex markets it seems necessary to introduce MS-type non-linearities.

4. Conclusions

This document seeks to contribute to the empirical literature by modeling and analyzing the volatility of returns in stock and Forex markets for HI countries and Latam EMEs. Using a sample of markets and countries, as well as a broad set of single-regime GARCH/GJR and MS-GARCH/MS-GJR models, this document pursues the following objectives: estimating and analyzing the behavior of the high-volatility regime while identifying events associated with stress periods; calculating the persistence of this regime; and identifying the presence of biases, heavy tails, and leverage effects based on the distributions selected for the estimations. The selection of the best models is done taking into account several criteria: value of the log-marginal likelihood, significance of the parameters; and evaluation of the smoothed probability curve for the high-volatility regime associated with the correct identification of the main domestic and internationals events involving volatility stress.

To our best knowledge, this is the first comparative work between a diverse group of HI countries and EMEs, as well as between stock and Forex markets, using a wide variety of single-regime GARCH/GJR and MS-GARCH/MS-GJR models with different innovations. The main results are: (i) the models selected using Ardia et al. (2018) have a better fit than those estimated by Augustyniak (2014), contain skewed distributions, and often require that the main coefficients be different in each regime; (ii) in Latam Forex markets, estimates of the heavy-tail parameter are smaller than in HI Forex and all stock markets; (iii) the persistence of the high-volatility regime is more significant and evident in stock markets (especially in Latam EMEs); (iv) in (HI and Latam) stock markets, a single-regime GJR model (leverage effects) with skewed distributions is selected; but when using MS models, virtually no MS-GJR models are selected. However, this does not happen in Forex markets, where leverage effects are not found either in single-regime or MS-GARCH models.

In most cases, persistence in the high-volatility regime is better captured using the ABBC method. This may be due to the fact that it allows the use of non-Gaussian distributions and is more flexible than AGK. At the same time, we have also found cases where AGK and ABBC estimates are very similar, and in some cases the estimates of the smoothed probabilities are better using AGK. While it is true that selecting periods with more persistence can help to identify high-volatility periods, in several cases the duration of the latter is long and difficult to validate. If this is not due to distributional flexibility, then it may be linked to the estimation method; i.e., while AGK addresses the problem of path dependence, ABBC do not. In this regard, an avenue of future research may be to extend the AGK method to allow flexible distributions and see if the resulting estimations make high-volatility regimes more persistent.

The results show distinctively different volatility behaviors and dynamics in stock and Forex markets and between HI countries and Latam EMEs. Latam markets appear to be more volatile and need heavier tails and biases in their distributions. This has substantial implications for modeling and calculating risk measures in these markets and countries. Therefore, a possible extension of this document is to evaluate the different models in terms of forecasting. The calculation of risk measures following the ABBC approach, as well as the Bayesian estimation of these models, is another issue for study.

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